**MDS: Home Work**

**9/10/2021**

1. Prove that ℝ*n* is a vector space, for all integers *n* ≥ 1.

2. Consider the following two matrices:



(a) Compute the third column of *AB* without computing the entire matrix *AB*.

(b) Compute the second row of *AB* without computing the entire matrix *AB*.

3. Confirm by matrix multiplication that the inverse of

4. Prove that the matrix  is

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5. Prove that the matrix  is not invertible if *ad* – *bc* = 0.

6. Show that if *A* is any matrix, then *ATA* and *AAT* are both symmetric.

7. For the following linear transformations, what must be the dimension of the corresponding matrix?

(a) *T* : ℝ2 → ℝ3

(b) *T* : ℝ3 → ℝ3

(c) *T* : ℝ4 → ℝ2

(d) *T* : ℝ4 → ℝ1

(e) *T* : ℝ1 → ℝ4

(f) *T* : ℝ1 → ℝ1

8. Is there a linear transformation *T* : ℝ3 → ℝ3 such that



If so, what is the matrix?

9. (a) What is the matrix of the linear transformation *S* : ℝ3 → ℝ3 corresponding to a reflection in the plane of the equation   
*x*1 = *x*2?

(b) What is the matrix T : R3 ! R3 corresponding to reection in the plane *x*2 = *x*3?

(c) What is the matrix *S* ⋅ *T*? What is the matrix *T* ⋅ *S*?

(d) What is the relationship between [*S* ⋅ *T*] and [*T* ⋅ *S*]? Here,   
[*S* ⋅ *T*] and [*T* ⋅ *S*] are the matrices representing the linear transformations *S* ⋅ *T* and *T* ⋅ *S* respectively.

10. Consider the transformation

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corresponding to a rotation by *θ* counterclockwise around the origin. Use compositions of this transformation to derive the fundamental theorems of trigonometry:

cos(*θ*1 + *θ*2) = cos(*θ*1) cos(*θ*2) – sin(*θ*1) sin(*θ*2)

sin(*θ*1 + *θ*2) = sin(*θ*1) cos(*θ*2) + cos(*θ*1) sin(*θ*2).

**Due Date: Nov 14, 2021**